

Finding Volume

In this unit, students find the volume of rectangular prisms and figures composed of two prisms.

Section A: Unit Cubes and Volume

In this section, students learn to call the amount of space an object takes up **volume**. Volume is measured in cubes. For example, this prism has a volume of 120 cubes.



To find the volume of any prism, students can find the number of cubes in one layer and multiply that number by the number of layers. In this example, students might describe this prism as having 5 layers of 24 cubes.

They can find the number of cubes by multiplying 5 and 24. So, $5 \times 24 = 120$.

Section B: Expressions for Finding Volume

In this section, students find the volume of a rectangular prism by multiplying the side lengths or by multiplying the area of the base by the height.



For example they can multiply the length by width by height, or $3 \times 5 \times 6$ or they can find the bottom area by multiplying 3×5 to get 15 and then multiplying 15 by 6. The volume of this rectangular prism is 90 cubic units.

Section C: Volume of Solid Figures

In this section, students learn that some figures are made from two rectangular prisms. They break apart these figures and find the volume of each prism. Then, they add the volumes of the two prisms to find the total volume of the figure.





Depending on how they break it apart, they can find the volume in different ways. They could multiply in these ways to find the volume of the figure:

 $(3 \times 3 \times 5) + (5 \times 2 \times 5)$ $(3 \times 5 \times 5) + (2 \times 2 \times 5)$

Try it at home!

Near the end of the unit, find the volume of these figures with your student.





- How are the 2 problems the same? How are they different?
- Can you explain or show me how you found the volume?
- How did you know you needed that number or piece of information?



Fractions as Quotients and Fraction Multiplication

In this unit, students solve problems involving division of whole numbers with answers that are fractions (which could be in the form of mixed numbers). They develop an understanding of fractions as the division of the numerator by the denominator, that is $a \div b = \frac{a}{b}$. They then solve problems that involve the multiplication of a whole number by a fraction or mixed number.

Section A: Fractions as Quotients

In this section, students learn that fractions are quotients and can be interpreted as division of the numerator by the denominator. Students draw and analyze tape diagrams that represent sharing situations. Through the context of first sharing 1, then sharing more than 1, then sharing a number of things with increasingly more people, students notice patterns and begin to understand that in general $\frac{a}{b} = a \div b$. For example, students use the diagram below to show 4 objects being shared by 3 people, or $4 \div 3$, which can also be written as a fraction, $\frac{4}{3}$.



Section B: Fractions of Whole Numbers

In this section, students make connections between multiplication and division and use visual representations that can show both operations. For example, the diagram above can also represent 4 groups of $\frac{1}{3}$, or $4 \times \frac{1}{3}$. Students discover ways of finding the product of a fraction and whole number that make sense to them and connect the product to the context and diagrams. They multiply a whole number by a fraction, $\frac{a}{b} \times q$.

Section C: Area and Fractional Side Lengths

In this section, students use what they know about the area of rectangles with whole number side lengths to find the area of rectangles that have one whole number side length and one fractional side length.



The expression 6×1 represents the area of a rectangle that is 6 units by 1 unit.



In the same way, $6 \times \frac{2}{3}$ represents the area of a rectangle that is 6 units by $\frac{2}{3}$ unit.



In addition, students see that the expressions $6 \times \frac{2}{3}$, $6 \times 2 \times \frac{1}{3}$, and $12 \times \frac{1}{3}$ can all represent the area of this same diagram.

Students analyze diagrams where one side length is a mixed number, for example a rectangle that is 2 by $3\frac{2}{5}$. They decompose the shaded region to show the whole units and the fractional units.



To find the area represented by this diagram, students may see two rectangles: a rectangle that is 2 units by 3 units and a rectangle that is 2 units by $\frac{2}{5}$ unit. While they may recognize that the area can be represented as $2 \times 3\frac{2}{5}$, students who see the decomposed rectangle may write $(2 \times 3) + (2 \times \frac{2}{5})$ to find the area.



Try it at home!

Near the end of the unit, ask your student the following questions:

1. Write as many expressions as you can that represents this diagram:

	4				
3 5					

2. What is the area of the following rectangle?



- How are the two problems similar? How are they different?
- How does your expression represent the diagram?
- How did you break up the rectangle to help you solve for the entire area?
- What are the side lengths of the rectangle?



Multiplying and Dividing Fractions

In this unit, students use area concepts to represent and solve problems involving the multiplication of two fractions, and generalize that when they multiply two fractions, they need to multiply the two numerators and the two denominators to find their product. They also reason about the relationship between multiplication and division to divide a whole number by a unit fraction and a unit fraction by a whole number.

Section A: Fraction Multiplication

In this section, students build on their knowledge of fraction multiplication developed in the previous unit by using area concepts to understand the multiplication of a fraction times a fraction. Students draw diagrams to represent the fractional area. For example, students learn that the diagrams below can represent the situation "Kiran eats macaroni and cheese from a pan that is $\frac{1}{3}$ full. He eats $\frac{1}{4}$ of the remaining macaroni and cheese in the pan. How much of the whole pan did Kiran eat?"



Students extend this conceptual understanding to multiply all types of fractions including fractions greater than 1 (for example, $\frac{7}{4}$). In each case, the students relate this multiplication to finding the area of a rectangle with fractions as side lengths. As the lessons progress, they notice that they can multiply the two numerators and the two denominators to find their product. This reasoning holds true for fractions greater than 1. For example, $\frac{3}{4} \times \frac{7}{5} = \frac{3 \times 7}{4 \times 5} = \frac{21}{20}$.



Section B: Fraction Division

The section begins by using whole numbers to recall that the size of the quotient depends, for example, on the amount being shared and the number of people sharing. That is, each student will get more pretzels if 3 students share 45 pretzels than if 3 students share 24 pretzels. Similarly, each student will get fewer pretzels if 6 students share 24 pretzels than if 3 students share 24 pretzels.

This thinking helps students understand why dividing a whole number by a unit fraction results in a quotient that is larger than the whole number. For example, $2 \div \frac{1}{3} = 6$ because there are 6 groups of $\frac{1}{3}$ in 2. As students draw diagrams and write expressions involving the division of unit fractions, students recognize the relationship between multiplication and division. For example, they may notice that $2 \div \frac{1}{3} = 6$ because $6 \times \frac{1}{3} = 2$, and that $\frac{1}{5} \div 2 = \frac{1}{10}$ is related to $2 \times \frac{1}{10} = \frac{1}{5}$.

Section C: Problem Solving with Fractions

In this section, students apply what they have learned in the previous sections through problem solving. Students see how fraction multiplication and division are useful in different contexts. They use the meaning of multiplication and division to decide which operation to use to solve various problems. As students share strategies, they may realize that some problems could be solved using either division or multiplication.

Try it at home!

Near the end of the unit, ask your student to solve the following question:

A painter was painting a wall yellow. He painted $\frac{1}{3}$ of the wall yellow before being told he needed to paint the wall blue. At the end of the day, he was able to cover up $\frac{1}{5}$ of the yellow wall in blue. How much of the entire wall is blue?

- Can you draw a diagram to help you solve the problem?
- What equation would you use to solve the problem?
- Can you solve this using division or multiplication instead?

Wrapping Up Multiplication and Division with Multi-Digit Numbers

In this unit, students multiply and divide multi-digit whole numbers using place value understanding, properties of operations, and the relationship between multiplication and division. They use the standard algorithm to multiply multi-digit whole numbers and partial quotients algorithms to divide whole numbers up to four digits by two digits. They then apply these skills as they solve problems involving volume.

Section A: Multi-digit Multiplication Using the Standard Algorithm

Students begin this unit by estimating products and quotients in a real-world context. Students use their understanding of place value, and their understanding of powers of 10 to make reasonable estimates. Students connect multiplication strategies, like partial products, to the standard multiplication algorithm. This is the partial products area diagram for 412×32 .

	400	10	2
30	30 × 400	30 × 10	30 × 2
2	2 × 400	2 × 10	2 × 2

They find partial products using area diagrams, and then translate that to a series of equations. These equations are compared against the steps in the standard algorithm to learn how the steps are based on place value reasoning and why the algorithm works. This table shows the connection between an algorithm using partial products and the standard algorithm.

Partial Products Area Diagram

Standard Algorithm

Area Diagram Aligned to Standard Algorithm



Section B: Multi-digit Division Using Partial Quotients

Students begin the work on whole number division by deepening their understanding of division expressions and the effect that changing the divisor or dividend has on the value of the

quotient. In a progression that leads to students engaging in algorithms using partial quotients, students estimate quotients and write partial quotient equations that match their own methods for finding the value of the quotient. Once students understand that they can find the value of the quotient by decomposing the dividend into multiples of the divisor, students learn to express this decomposition using equations and then an algorithm using partial quotients.

Decomposition of the Dividend	An Algorithm Using Partial Quotients	
$448 \div 16 = (320 \div 16) + (80 \div 16) + (48 \div 16)$	28	
$448 \div 16 = 20 + 5 + 3$	3	
$448 \div 16 = 28$	5 20	
	16)448 —320 (20 × 16)	
	128	
	$-80 (5 \times 16)$	
	<u>– 48</u> (3 × 16)	
	0	

Section C: Let's Put it to Work

Students practice their multiplication and division skills as they solve problems involving volume. Students are using the volume formulas ($V = l \times w \times h$ and $V = b \times h$) to practice the multiplication and division work of the previous sections. Students engage with relatively large numbers to multiply and divide using these volume formulas, developing fluency with the standard algorithm for multiplication and the algorithm using partial quotients.

Try it at home!

Near the end of the unit, ask your student to solve the following problems:

- 219 × 52
- 868 ÷ 14

- Can you draw a diagram to help you solve the problem?
- Can you explain the steps of your algorithm?



Place Value Patterns and Decimal Operations

In this unit, students use place value understanding to round, compare, order, add, subtract, multiply, and divide decimals.

Section A: Numbers to Thousandths

In this section, students are introduced to the thousandths place. They represent decimals on gridded area diagrams where the large square has a value of 1, and each small square within represents $\frac{1}{100}$.

Students learn that if they partition each small square into ten equal parts, each of those parts represents 1 thousandth of the large square.



Students write decimals in expanded form using sums of multiplication expressions. For example, 0.124 in expanded form can be written as $(1 \times \frac{1}{10}) + (2 \times \frac{1}{100}) + (4 \times \frac{1}{1,000})$.

Students use this developing understanding of place value to the thousandths to locate decimals on a number line. They then use the number line to round, compare, and order decimals.

Section B: Add and Subtract Decimals

In this section, students add and subtract decimals to the hundredths. Initially, students add and subtract in ways that make sense to them. This allows students to relate addition and subtraction of decimals to operations with whole numbers. Students also use place value reasoning to estimate the value of sums and differences.



Section C: Multiply Decimals

In this section, students multiply decimals with products up to hundredths. Students initially multiply decimals in ways that make sense to them. Area diagrams were used to make sense of fraction multiplication in earlier units, and they are used here as a familiar representation to make sense of decimal multiplication. They use the diagrams to relate multiplying with whole numbers to multiplying with decimals.

For example, the diagram shown can represent 2 groups of 6 hundredths, or 12 hundredths, which can be written as the equation $2 \times 0.06 = 0.12$.

Students may also see this as 2 times 6 groups of 1 hundredth or $2 \times 6 \times 0.01 = 12 \times 0.01 = 0.12$.

To multiply tenths by tenths, students revisit area concepts from previous units. Using area diagrams, they find the area of the shaded region by multiplying side lengths. They use decimal notation to mark the side lengths. The diagram shows how the students would represent 0.3×1.4 to arrive at an answer of 0.42.

Section D: Divide Decimals



Just as with whole numbers and fractions, students use the relationship between multiplication and division to make sense of division with decimals. In this section, students consider how many tenths or hundredths are in whole numbers (that is, 10 tenths are in 1 whole, 100 hundredths are in 1 whole). This understanding provides a foundation for students to divide a whole number by any amount of tenths or hundredths. Students learn how to use diagrams to help them solve division problems.



The example shows how students can divide 4 into groups of 2 tenths. There are 20 groups of 2 tenths in 4 wholes.

$$4 \div 0.2 = 20$$



Try it at home!

Near the end of the unit, ask your student to solve the following problems:

- 1.8 × 0.2
- 12.1 ÷ 1.1

- Can you draw a diagram to help you solve the problem? How does your diagram show the solution?
- Can you explain the steps of your algorithm?



More Decimal and Fraction Operations

In this unit, students solve multi-step problems involving measurement conversions, line plots, and fraction operations, including addition and subtraction of fractions with unlike denominators. They also explain patterns when multiplying and dividing by powers of 10. Students interpret multiplication as scaling by comparing products with factors.

Section A: Measurement Conversions and Powers of 10

In this section, students convert smaller units to larger units (for example, centimeters to kilometers), and describe the patterns they notice when multiplying and dividing by powers of 10. Students work with the metric and customary system (for example. feet, quarts, pounds, and so on) and develop an understanding of the relative sizes of units of length, volume, and weight. Students use the four operations with whole numbers, decimals, and fractions to solve multi-step word problems involving measurement conversions.

Section B: Add and Subtract Fractions with Unlike Denominators

In this section, students add and subtract fractions and mixed numbers with unlike denominators, and apply this learning to problem solving. Students first encounter problems where one denominator is a factor of the other (for example, $\frac{1}{4}$ s and $\frac{1}{8}$ s), so that they will only need to change one denominator. Then, students solve problems where the denominators are not related (for example, $\frac{1}{3}$ s and $\frac{1}{4}$ s). Students conclude that multiplying the denominators or finding a common multiple are helpful ways to create common denominators.

Students also extend their understanding of line plots. They create line plots using measurement data in fractional units (halves, fourths, and eighths), and interpret the data on line plots to solve problems involving the four fraction operations like this one.

Jada says $\frac{3}{4}$ of the students spend less than 2 hours on a screen. Is she correct?



Explain how you know your answer is correct.



Section C: The Size of Products

In this section, students build on their understanding of multiplication to include the concept of scaling. Students interpret multiplication expressions as a quantity that is resized or scaled by a factor.

Students compare multiplication expressions without performing the multiplication. In the example shown, students reason that $\frac{7}{6} \times 4$ is greater than the other two expressions because in each expression, 4 is being multiplied by a fraction, and $\frac{7}{6}$ is the largest fraction of the three.

Which of these expressions represents the largest product?

$$\frac{5}{8} \times 4 \qquad \qquad \frac{7}{6} \times 4 \qquad \qquad \frac{1}{2} \times 4$$

Students locate multiplication expressions on a number line, and analyze expressions to determine if the product is greater than, less than, or equal to one of its factors. Students make sense of their learning by recognizing that if a given number is multiplied by:

- a fraction greater than 1, then the product will be greater than the given number
- a fraction less than 1, then the product will be less than the given number
- a fraction equal to 1, then the product will be equal to the given number

Try it at home!

Near the end of the unit, ask your student to solve the following problems:

- How many kilometers is equal to 200 centimeters?
- $\frac{2}{3} + \frac{2}{9}$
- $\frac{2}{3} + \frac{5}{8}$
- Will $\frac{4}{3} \times 5$ be greater than, less than, or equal to 5? How do you know?

- What strategy are you going to use to help you solve the problem?
- Could you have solved the problem in a different way?
- Which problem was easier to solve? Why?



Shapes on the Coordinate Plane

In this unit, students are introduced to the structure of the coordinate grid, and the convention and notation of coordinates to name points. They classify triangles and quadrilaterals in a hierarchy based on properties of side length and angle measure. In their work with numerical patterns, students generate two different numerical patterns, and identify relationships between the corresponding terms in the patterns.

Section A: The Coordinate Plane

In this section, students explore the coordinate grid.

- They recognize that a point is located where two lines intersect.
- They describe points on the grid based on the numbers on the horizontal and vertical axes.

For example, the point shown is located at (7, 3).



Section B: The Hierarchy of Shapes

In this section, students learn more about shapes. They sort different types of triangles and quadrilaterals based on what the shapes have in common. They classify the shapes into categories and subcategories. For example,





Section C: Numerical Patterns

In this section, students generate patterns and explore relationships between patterns. For example:



Compare your patterns. What relationships do you notice?

After students become familiar with generating patterns from rules and explaining relationships between patterns, they plot pairs of numbers from two patterns on a coordinate grid. They also represent and solve problems by graphing points on the coordinate grid.

Try it at home!

Near the end of the unit, ask your student to solve the following problem:

This coordinate grid represents information about rectangles A–D. Based on the coordinate grid, what do we know about each of these rectangles?

- What strategy are you going to use to help you solve the problem?
- How can you show the rectangles represented by these points on the grid?
- Add another point to the grid that represents a different rectangle and describe the rectangle to me.





Putting It All Together

In this unit, students put together their understanding from throughout the year to cap off major work and fluency goals of the grade.

Section A - Multiply and Divide Whole Numbers

Students deepen their understanding of the standard algorithm for multiplication and practice using it. They recognize and explain place value patterns when multiplying multi-digit numbers, learn how to use the algorithm when one or more of the factors has several zeroes, and examine an alternative recording method for newly composed units.

Section B - Apply Volume Concepts

Students investigate volume by calculating the volume of rectangular prisms and estimating the number of books in a library. Students solve problems in a real-world context and consider what it means to measure volume using different units. They estimate the amount of water that falls on the roof of their home and use it to reason whether or not that amount is sufficient for many of the daily household chores that use water.

Section C - Fraction and Decimal Operations

Students operate with fractions and decimals. Each lesson is structured as a game day where students learn games for adding and subtracting fractions, adding and subtracting decimals, and multiplying fractions.

Section D - Creation and Design

Students have the opportunity to apply what they've learned about instructional routines throughout the year to create their own Notice and Wonder, Exploration Estimation, Number Talk, True or False, and Which One Doesn't Belong. Students design and create routines in small-groups and can facilitate their routine with another group in the class. Each lesson has a content focus provided, so students review major work of the grade through their routines.



Try it at home!

Near the end of the unit, ask your student to share the instructional routines they created. Questions that may be helpful as they share:

- How did you design the routine?
- How does the routine relate to what you learned this year?
- How could we modify the routine?