Family Support Materials

Sequences and Functions

In this unit, your student will be remembering ways to represent functions. In mathematics, we can think of a function as a rule that tells us how to go from an input to an output. A *sequence* is a special type of function in which the input is a position in a list, and the output is the number in that position. If you have ever used "fill down" to continue a pattern in a spreadsheet, you have created a sequence. For each sequence of numbers, can you guess a possible rule for creating the next number?

Sequence A: 4, 7, 10, 13, ________

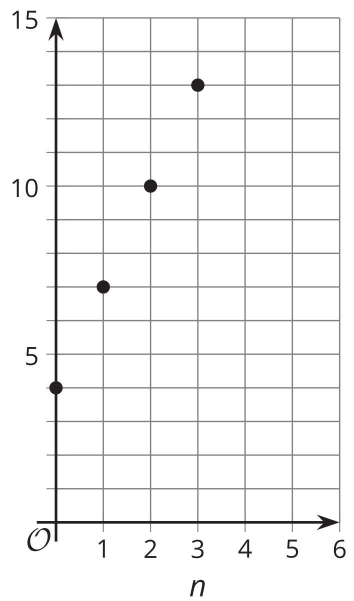
Sequence B: 2, 6, 18, 54, ________

You probably noticed that a rule for Sequence A could be "add 3 to any term to get the next term." There are different ways we could represent this sequence.

Using a table:

position in list	0	1	2	3	n
term	4	7	10	13	$4 + 3 \times n$

Using a graph:



Using words:

"To find the nth term, multiply n by 3 and add 4."

Using notation for defining a function:

$f(n) = 4 + 3 \times n$ (the value of the nth term is $4 + 3 \times n$). For example, $f(2) = 4 + 3 \times 2$, so $f(2) = 10$ (the value of the 2nd term is 10).

Here is a task to try with your student:

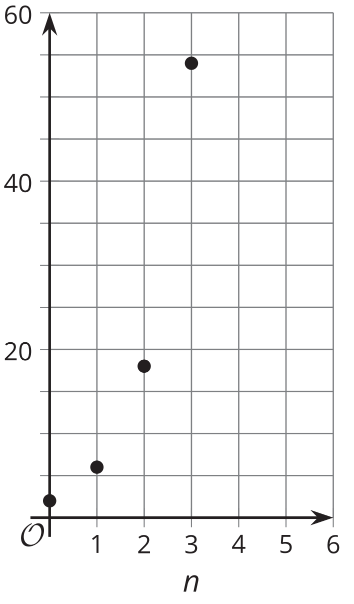
Let's revisit Sequence B: 2, 6, 18, 54, . . .

1. Describe any patterns you notice.
2. If the pattern is "multiply any term by 3 to get the next term," what is the next term?
3. If we call 2 the "0th term," what is the 10th term?
4. How could we express the nth term?
5. Represent Sequence B in as many different ways as you can.

Solution:

1. It is possible to describe many patterns in this list.
2. 162
3. 118,098
4. 2×3^n. This can also be written $2(3^n)$ or $2 \cdot 3^n$.
5. Here are some ways:

position in list	0	1	2	3	n
term	2	6	18	54	2×3^n



"Multiply any term by 3 to get the next term."

$f(n) = 2 \times 3^n$

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Polynomials and Rational Functions

In this unit, your student will learn about a kind of function, *polynomials*. (In earlier grades, students learned about two special kinds of polynomial functions: linear and quadratic functions.) A polynomial is a sum of terms involving only one letter, called a variable, where the exponents of the variable are whole numbers. For example, $3x^3 - x^2 + 10$ and $5x^6$ are polynomials. But $6x^{-2} + 2x^{-1}$ is not, because the exponents are negative. And $2xy - 7x$ is not, because it involves more than one variable. Your student will connect different ways of representing polynomial functions, such as graphs and equations.

Multiplication and division of numbers will be extended to polynomials, so this is a good time to refresh skills with multiplying and dividing numbers by hand. When numbers are multiplied, we often use the distributive property, so that each piece of one number is multiplied by each piece of the other number. For example, 34 is 30 plus 4, or 3 tens plus 4 ones. The tens and ones of each number are multiplied by the tens and ones of the other, and then all the results are added. When polynomials are multiplied, we also use the distributive property. Here is an example of each:

$$\begin{aligned}&(30+4)(10+5)\\&=30(10+5)+4(10+5)\\&=30\cdot 10+30\cdot 5+4\cdot 10+4\cdot 5\\&=300+150+40+20\\&=510\end{aligned}$$

$$\begin{aligned}&(x-7)(2x+3)\\&=x(2x+3)+(-7)(2x+3)\\&=x\cdot 2x+x\cdot 3+(-7)\cdot 2x+(-7)\cdot 3\\&=2x^2+3x-14x-21\\&=2x^2-11x-21\end{aligned}$$

Multiplication, with numbers or polynomials, can be represented in lots of ways, and your student should find a way that makes sense and is useful. Ask your student to show you how to multiply polynomials.

Long division with polynomials looks a lot like long division with numbers. Here is an example of each:

Division can also be represented in many ways, so if you or your student learned a different way of doing long division, that way can also be extended to polynomials.

Here are some tasks to try with your student:

1. Multiply 47 by 25, using any method you like. Try using that same method to multiply $(4x+7)(2x+5)$. What was the same? What was different?

2. Divide 372 by 12, using any method you like. Then represent the division another way, for example using pictures or words.

3. Factor these expressions. Check your answers by multiplying the factors. When you were factoring and multiplying, how did you know what to do at each step?
    a. $x^2 + 5x + 6$

    b. $x^2 + 2x - 8$

Solution:

1. One way to multiply 47 by 25 is to use a standard multiplication algorithm. We can do something similar with $(4x + 7)(2x + 5)$. Just as we multiplied 47 by 5 and then by 20 and then added the results together, we can multiply $4x + 7$ by 5 and then by $2x$ and then add the results together. Here are the two versions:

$$\begin{array}{r} 47 \\ \underline{\times 25} \\ 235 \\ \underline{+940} \\ 1175 \end{array} \qquad\qquad \begin{array}{r} 4x + 7 \\ \underline{\times\ \ 2x + 5} \\ 20x + 35 \\ \underline{+\ \ 8x^2 + 14x + \ \ 0} \\ 8x^2 + 34x + 35 \end{array}$$

2. One way to divide 372 by 12 is the standard division algorithm (shown earlier). Another way to do it is by subtraction. To be more efficient, we can take away groups of 120 (ten 12's) until the result is less than 120, and then take away groups of 12. We can take away three groups of 120 and 1 group of 12 from 372, and then we have nothing left over. So there are 31 groups of 12.

3.
    a. $x^2 + 5x + 6 = (x + 3)(x + 2)$

    b. $x^2 + 2x - 8 = (x + 4)(x - 2)$

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Complex Numbers and Rational Exponents

In this unit, your student will extend what they know about numbers and exponents. They will use familiar exponent rules to see how to evaluate expressions with exponents that are fractions, like $5^{2/3}$. They will also use what they know about quadratic functions and square roots to learn about a new kind of number: imaginary numbers. Imaginary numbers are multiples of the square root of -1 (also known as i). So far, your student has used only real numbers, and no real number can square to make -1.

Square and cube roots will be studied in depth in this unit. Starting from the geometric meaning of square and cube roots, your student will learn to solve equations with variables inside square and cube roots. In geometry, roots are connected to area and volume. For example, if a square has an area of 16 ft^2, then each of its sides is 4 feet long, because 4 is the square root of 16. If a cube has a volume of 8 in^3, then each of its edges is 2 inches long, because 2 is the cube root of 8.

Here are some tasks to try with your student:

1.
   a. If a square has sides that are 5 feet long, what is the area of the square?

   b. If another square has an area of 20 ft^2, about how long is each of its sides? Try to find an estimate without using a calculator, then check to see how close your estimate was. What would be a better estimate?

2.
   a. If a cube has edges that are 3 meters long, what is its volume?

   b. If another cube has a volume of 30 m^3, about how long is each of its edges? Estimate without a calculator, then check to see how close your estimate was. What would be a better estimate?

3.
   a. If $m^2 = 4$, what could m be? Explain how you know.

   b. If $k^2 =$ -4, what could k be? Explain how you know.

Solution:

1.

a. 25 ft^2.

b. A little less than 5 ft, so maybe 4.8 ft. If I square 4.8, I get 23.04, so 4.8 is too big. A better estimate would be 4.5, which squares to make 20.25.

2.

a. 27 m^3.

b. A little larger than 3 m, so maybe 3.25 m. If I cube 3.25, I get about 34.33, so 3.25 is too big. A better estimate would be 3.1, which cubes to make 29.791.

3.

a. m could be 2, because $2 \cdot 2 = 4$. But m could also be -2, because $\text{-}2 \cdot \text{-}2$ is also 4.

b. I don't think there's anything k could be. If it's positive, then squaring it will give us a positive number, but if it's negative, then its square will also be positive.

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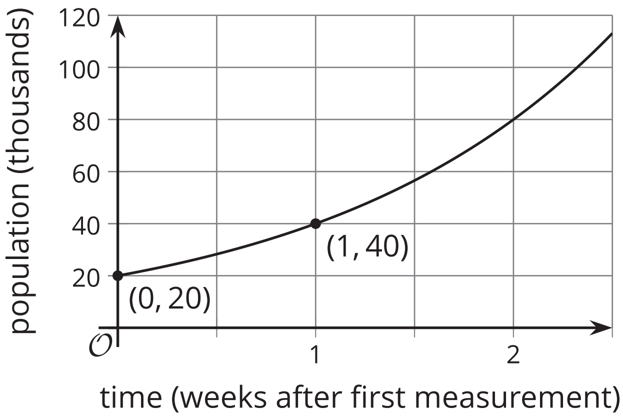
Exponential Functions and Equations

In this unit, your student will look at exponential functions and use them to solve problems. Exponential functions are used to model many real-world situations. For example,

- Many populations grow exponentially, especially when resources are readily available.
- Contagious diseases can spread exponentially when first introduced to a population.
- Radioactive substances, like those used in medical treatments or nuclear power plants, decay or decrease exponentially in predictable ways.

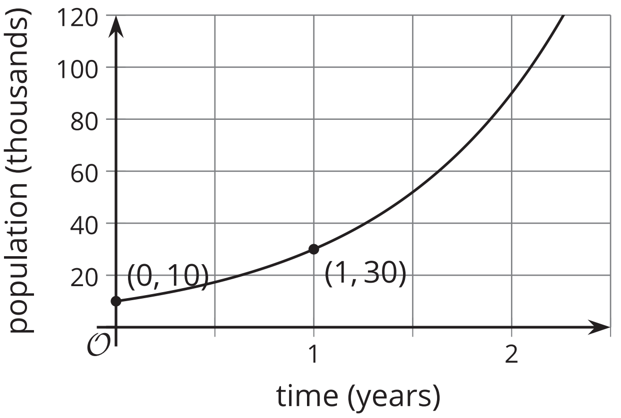
Here is a graph showing an insect population p, in thousands, w weeks after it was first measured.



The population is growing exponentially, doubling each week. An equation relating p and w is $p = 20 \cdot 2^w$. But what if we want to see how quickly the insect population grows each day? Because the growth is exponential, we know it grows by the same factor each day. If one week of growth means multiplying by 2, then one day of growth means multiplying by the seventh root of 2, $2^{\frac{1}{7}}$, since this is the number whose seventh power is 2. Using this factor, if d is the number of days since the insect population was measured, the relationship between p and d is $p = 20 \cdot \left(2^{\frac{1}{7}}\right)^d$. Now we have an equation we can use to estimate the population by days instead of by weeks.

Here is a task to try with your student:

Here is the graph of a different exponentially increasing population a, in thousands, given by the equation $a = 10 \cdot 3^t$. Here t is time measured in years.



1. What do the labeled points $(0, 10)$ and $(1, 30)$ mean in this situation?
2. By what factor does the population grow each month? Hint: how can you use the number of months in a year to express this factor?
3. Write an equation for the population, in thousands, m months after it was first measured.
4. After about how many months did the population reach 50,000?

Solution:

1. The point $(0, 10)$ means that the population was 10,000 when first measured and was 30,000 after 1 year.
2. $3^{\frac{1}{12}}$
3. $p = 10 \cdot \left(3^{\frac{1}{12}}\right)^m$
4. between 17 and 18 months

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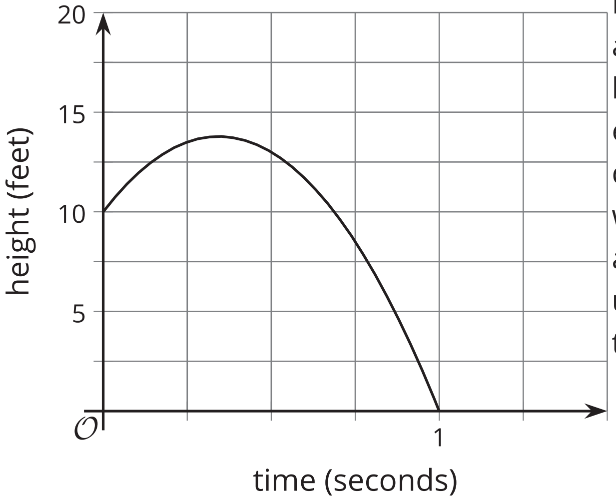
Transformations of Functions

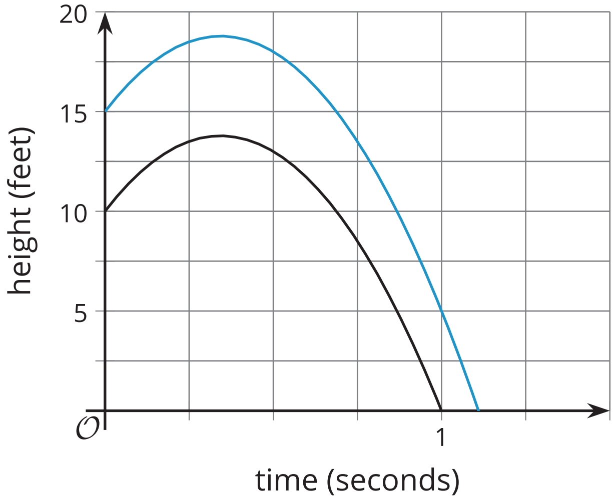
In this unit, your student will move graphs of functions around the plane and figure out how to write new functions representing these graphs. Many professions use functions to model real-world relationships. For example, an economist might study the relationship between price and revenue. An engineer might study the relationship between temperature and efficiency of an engine. A psychologist might study the relationship between screen time and anxiety. Analyzing changes to a graph representing a relationship can help people understand changes in the real-world relationship being modeled.

For example, here is a graph representing the height of a diver over the water after jumping from a diving board.



If h represents the diver's height t seconds after jumping, an equation for the diver's height is $h = 10 + 22t - 32t^2$. In the equation, the 10 gives the height of the diving board, which is where the diver is when $t = 0$. The $22t$ term and the -$32t^2$ term account for the effects of the diver jumping up and gravity pulling the diver down toward the water.

What would the graph look like if the diver made the same jump off a diving board that was 15 feet above the water instead of 10 feet?

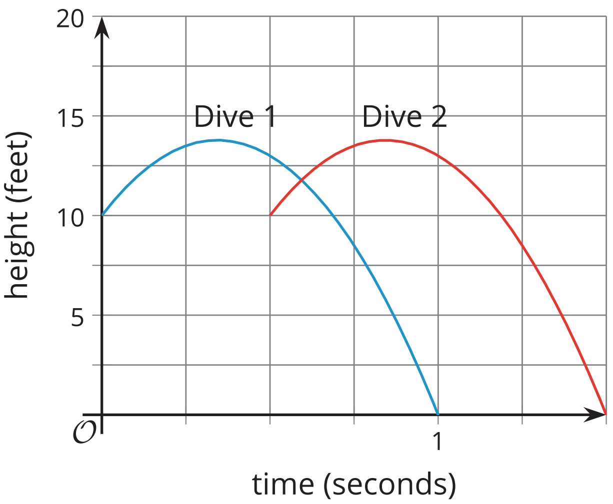


Notice that the graph is moved upward by 5 units. Instead of starting at 10 feet above the water, the diver starts at 15 feet. Instead of a maximum height of close to 14 feet, the maximum height is now close to 19 feet. An equation for the new graph is $h = 15 + 22t - 32t^2$. Notice that only the constant term changed: the 10 increased to 15.

Here is a task to try with your student:

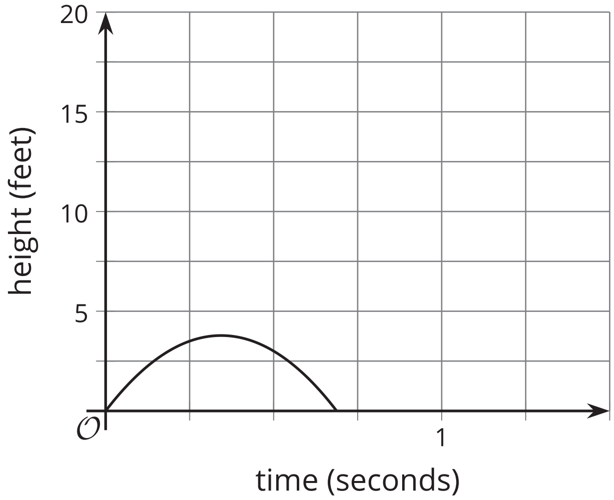
Let's look again at the diver's height represented by the equation $h = 10 + 22t - 32t^2$.

1. If the diver were to make the same jump starting at the level of the water, what equation would give her height?
2. Sketch a graph representing your equation, either by hand or using technology.
3. Use your graph to estimate when the diver would hit the water.
4. When does the diver reach the highest point in the dive? How does this compare to the high point in the dive when the diver jumps from 10 or 15 feet over the water?
5. Here is the graph of the equation $h = 10 + 22t - 32t^2$, labeled Dive 1, and a second graph for a different dive, labeled Dive 2. How do these two dives compare?



Solution:

1. $h = 22t - 32t^2$.

2. 

3. About $\frac{2}{3}$ of a second

4. Between $\frac{1}{4}$ and $\frac{1}{2}$ second, about $\frac{1}{3}$ of a second. This is the same time the diver was at the highest point in the other graphs too: the shape of the graph is the same just shifted vertically.

5. For each of the two dives, the diver starts from 10 feet and reaches a maximum height of close to 14 feet. In the second dive, the diver leaves the diving board a half second later than the diver in the first dive.

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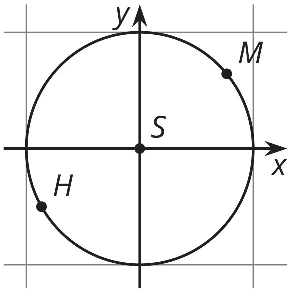
Trigonometric Functions

In this unit, your student will learn about periodic functions. These types of functions have a special feature: their output values repeat over and over and over again. This is a feature that none of the other functions students have studied with changing outputs up to now have, and it is a type of function students need if they want to model situations involving circular motion or other relationships where the same values repeat over and over again.

For example, consider the orbit of Mars around the sun, which can be modeled by a circle. Once every 687 days Mars completes a full circle and we say that the orbit of Mars has a period of 687 days. Here is a very simple sketch of the orbit of Mars, M, with an x- and y-axis centered on the Sun, S:



Using the period, we know that every 687 days Mars will be at the point marked M. We can also say, since Mars' speed is pretty constant, that 343.5 days later Mars will be at point H since that is half the period. Using different increments of the period we could predict the location of Mars at different points in its orbit throughout the Martian year.

Here are some other things that can be modeled by periodic functions:

- height off the ground at different rotations while riding a Ferris wheel
- average daily temperatures in a city over a year
- the position of a pendulum
- traffic congestion at a particular location

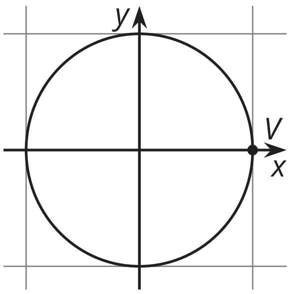
Here is a task to try with your student:

Venus' orbit has a period of about 225 days.

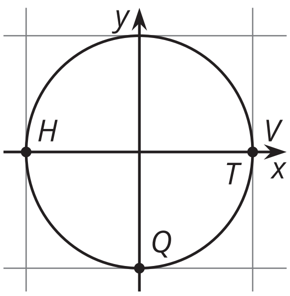
1. About how many orbits has Venus completed after 450 days?
2. About how many orbits has Venus completed after 365 days?
3. Use the simple sketch of Venus' orbit and the starting point marked V to plot Venus' location after different numbers of days. Assume Venus is rotating counterclockwise around the circle.
   a. 112.5 days (H)

   b. 168.75 days (Q)

   c. 2925 days (T)



Solution:

1. Venus completes 2 full orbits of the Sun in 450 days.
2. Venus completes 1 full orbit and is a bit over half way (62%) through its next orbit.
3. 

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Statistical Inferences

In this unit, your student will use a small *sample* of data to estimate information about a larger group called a *population* and use simulation to determine a range of values for the estimate. A population is the entire set of subjects of interest for a question and a sample is a smaller group within that population.

For example, we may want to figure out the *mean* (or average) amount that families in the United States spend on food every month. The population includes all families in the United States, but collecting information from everyone would be very difficult and would cost a lot of money, so we might begin gathering data with a sample of 50 families.

An important question to consider when beginning to collect information from a sample is how the sample is to be selected. The data you collect might be very different if you ask families who are shopping at a local grocery store compared to asking people outside of a fancy restaurant. Similarly, the amount spent on food in San Francisco is likely very different from the amount spent in rural Iowa. There may even be some spending habits that are hidden in ways we haven't thought about yet. So, how do you make sure that your sample is representative of families in the United States without using too many families from groups that are not typical in their spending?

The solution is to use *randomness*. We can select 50 families using a random process such as having a computer select the families from a database at random without considering other factors. This should reduce the *bias* that might be introduced by humans trying to get information about people and it will likely include more accurate proportions of the different types of families in the United States. While randomness may not entirely eliminate bias from the sample selection, it will significantly reduce the bias present when compared to selection without randomness.

Researchers have done studies like these and have found that the mean amount spent on food each month. A report says that the mean amount spent on food each month is $600 with a *margin of error* (MoE) of $150. The margin of error is used to say that the we don't expect every family in the sample to spend exactly $600.

The margin of error is important to look for in statistical results. It is irresponsible to discuss statistics without providing a margin of error to describe how much the value is expected to vary. Many graphs included in news reports will report it in small print on the graphic. Look for something like $\pm 3\%$ when there is a graphic about approval rating for an official or polls during the next election. This means that the percentages shown in the graphic could actually be up to 3% lower or higher than the number shown.

Here is a task to try with your student:

A town has an upcoming vote about whether to raise corporate income taxes by 2% to increase funding for public schools. The local news shows an image that indicates that 52% of the voting population are in favor of the tax increase and in the corner it shows "margin of error $\pm 3.5\%$." The reporter sounds confident that the corporate taxes will be increased because anything greater than 50% of the vote in favor of the taxes will pass the law.

1. The reporter who found the 52% number arrived at it by driving to 4 of the 20 different neighborhoods around town and asking residents their opinion. Is there anything wrong with how that was done? Can you think of a better way to collect data?

2. What does the margin of error mean in this image?

3. Should you be confident that the taxes will be increased? Explain your reasoning.

Solutions:

1. Going only to 4 neighborhoods in the town might leave out the opinions of many voters from other neighborhoods where the reporter did not go. A better way to collect information could be to randomly select several households in the town to survey about their opinion. The random selection is more likely to avoid any biases the reporter has about which neighborhoods to visit.

2. The margin of error means that the actual percentage in favor of increasing taxes could be 3.5% higher or 3.5% lower than the 52% reported based on the sample. This means that the actual percentage would fall between 48.5% and 55.5%.

3. Sample responses:
    - I think there is still a good chance the taxes will be increased. Although the actual percentage could be as low as 48.5% based on the margin of error, it could also be as high as 55.5%. Most of the possible percentages are above 50%, so I think the increase will happen.

    - I think it is still unclear whether the increase will happen. Based on the margin of error, the actual percentage could be as low as 48.5% which would cause the increase not to happen. I'm also not sure about the reporter's methods for collecting a sample in this report, so the report may not be very accurate.