Traditional Column-Addition:

Example: 248 + 187

Step 1:
```
  1
 248
+187
 ---
   5
```

Step 2:
```
 1 1
 248
+187
 ---
  35
```

Step 3:
```
 1 1
 248
+187
 ---
 435
```

Partial-Sums Addition:

Example: 248 + 187

		100's	10's	1's	
		2	4	8	
		+1	8	7	
Step 1:	Add ones		1	5	(8+7=15)
Step 2:	Add tens	1	2	0	(40+80=120)
Step 3:	Add hundreds	+3	0	0	(200+100=300)
Step 4:	Add all	4	3	5	

Note: This method uses what each digit is actually worth (place value) to add. This makes more logical sense to some students than "carrying" a number to the next column simply because they were told to do so (without understanding why). The **bold** part to the right does not need to be written unless the child wants to write it in order to stay organized.

Traditional Multiplication:

Example: 26 x 34

+2
Step 1: 26
x 34
4

+2
Step 2: 26
x 34
104

■ ← Cross out or erase
Step 3: 26
x 34
104
0 (place holder)

+1
■
Step 4: 26
x 34
104
80

+1
■
Step 5: 26
x 34
104
780

Step 6: 26
x 34
104
+780
884

Partial-Products Multiplication:

Example: 26 x 34

	10's	1's	
	2	6	
	x 3	4	
Step 1:	2	4	(4x6=24)
Step 2:	8	0	(4x20=80)
Step 3:	1 8	0	(6x30=180)
Step 4:	6	0 0	(20x30=600)

Step 5: Add each part of the product together.

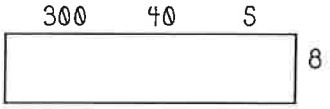
+1
24
80
180
+600
884

Note: This method uses what each digit is actually worth (place value) to multiply. This makes more logical sense to some students than "carrying" a number to the next column to add on (not multiply) without understanding why it is done. They also don't have to worry about the "place holder" zero that doesn't make sense to most kids and is often forgotten. The **bold** part to the right does not need to be written unless the child wants to write it in order to stay organized.

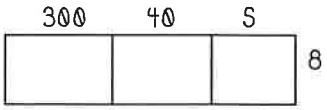
Area Model Multiplication (1x3):

Step 1: Draw a box long & wide enough for each place value (give lots of space). Example: 345 x 8

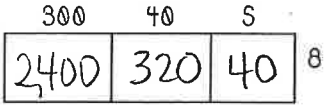
Step 2: Decompose each number by it's place value and write the expanded notation along the top and side of the box (I usually do the longer number on top but it does not matter).



Step 3: Draw enough boxes so each number on top has it's own column and each number on the side has it's own row.



Step 4: Multiply the numbers on the outside of each square by the number on the side (similar to using a multiplication table) and put the product inside the box.



Step 5: Find the sum of all of the boxes. (I usually add the biggest & smallest place values mentally and then add the others in between.)

2400 + 320 + 40

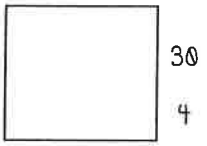
biggest smallest

2440		345
+ 320	→	x 8
2,760		2,760

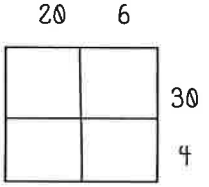
Area Model Multiplication (2x2):

Step 1: Draw a box long & wide enough for each place value (give lots of space). Example: 26 x 34

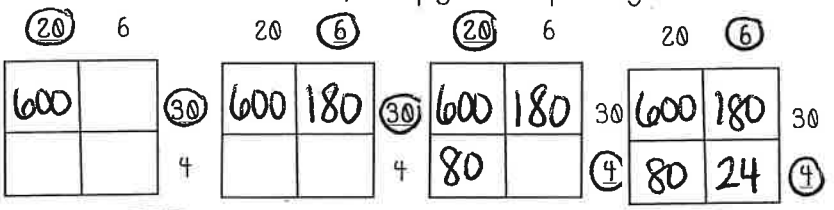
Step 2: Decompose each number by it's place value and write the expanded notation along the top and side of the box.



Step 3: Draw enough boxes so each number on top has it's own column and each number on the side has it's own row.



Step 4: Choose "upstairs" or "downstairs" to multiply first. For instance, if you choose "upstairs" (the 30), multiply each # on top by 30 (line up each answer just like using a multiplication table). Do the same with the "downstairs" (the 4), multiply both top #'s by 4.



Step 5: Find the sum of all of the boxes. (I usually add the biggest & smallest place values mentally and then add the others in between.)

600 + 180 + 80 + 24

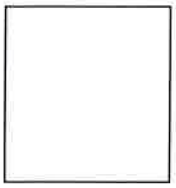
biggest smallest

624		26
+ 260	→	x34
884		884

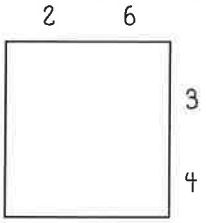
Lattice Multiplication: Example: 26 x 34

This method seems to be the most liked by students & disliked by parents. This way does take a little bit more writing, but I think kids like this way because it appears to be more of a puzzle. In addition, to do lattice they just have to know the basic facts without randomly carrying numbers to add later or using a "place holder" zero (like the traditional method) & they don't have to know extended multiplication facts or place value (like the partial products method). This is exactly like the area model, but without the zeros (and extended multiplication).

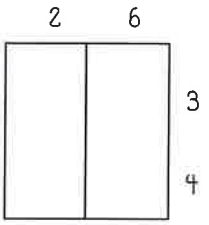
Step 1: Draw a box. ⇩



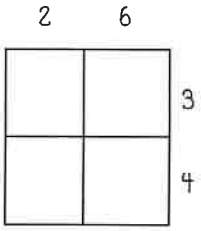
Step 2: Write the numbers along the box. It does not matter which number is along the top or side.



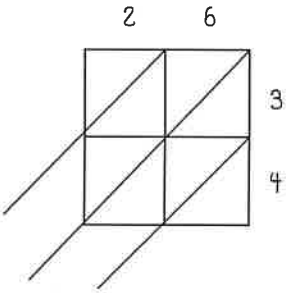
Step 3: Draw enough columns so each digit on top has it's own column.



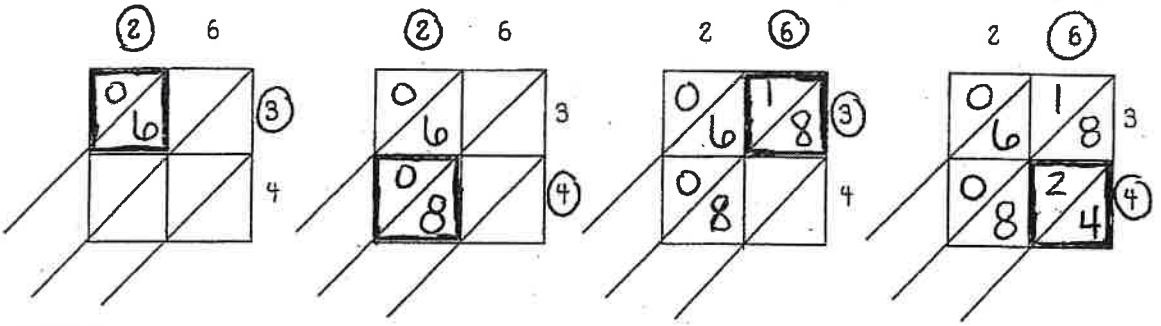
Step 4: Draw enough rows so each digit on the side has it's own row.



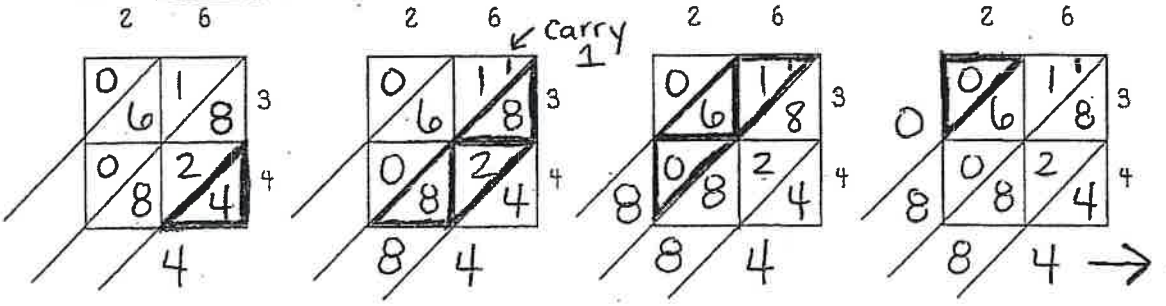
Step 5: Draw diagonals through each box by going from the upper right corner to the lower left corner. (I tell them each triangle should look like half of a sandwich.)



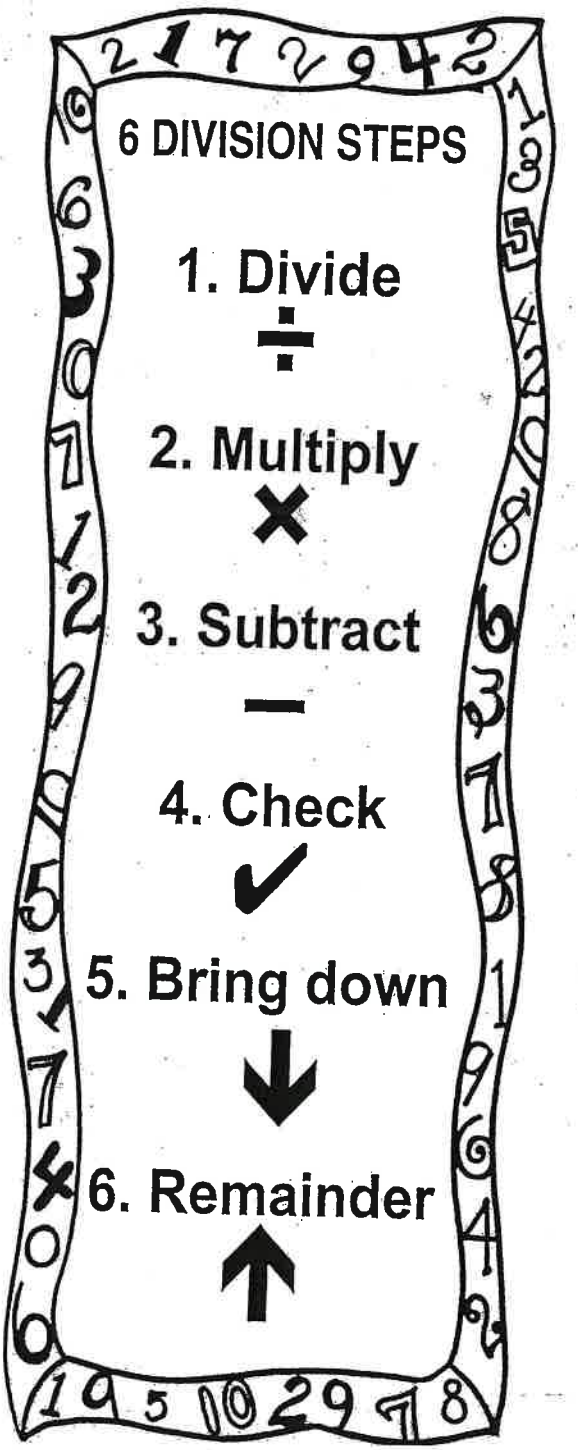
Step 6: Choose "upstairs" or "downstairs" to multiply first. For instance, if you choose "upstairs" (the 3), multiply each digit on top by 3 & make sure the answer goes under one of the numbers you multiplied and beside the 3. Do the same with the "downstairs" (the 4), multiply both top numbers by 4.



Step 7: Add all triangles in a diagonal row, starting with the right side. Carry numbers if needed.



$$\begin{array}{r} 26 \\ \times\ 34 \\ \hline 884 \end{array}$$



1. Divide – Ask yourself "How many groups of ___ (the divisor on the outside) can you make out of ___ (dividend on the inside)?"

2. Multiply – Multiply the answer you wrote on top times the number you are dividing by (the divisor).

3. Subtract – Subtract the product (answer from multiplying), from the place value you are working on, to see how much is left to divide.

4. Check – Ask yourself "Can I make any more groups of ___ (the divisor on the outside) out of what I have left?" The answer should always be NO, otherwise you have calculated something wrong.

5. Bring Down – Bring down the next place value & start dividing again at step #1.

6. Remainder – Anything that is left over (can no longer be divided) is the remainder. A remainder can be written as a fraction by putting the remainder on top and the divisor (the number on the outside) on the bottom OR a decimal by adding a decimal point and zeros to the original dividend (number inside) and continuing the division steps to at least the tenths place (usually hundredths).

You know you are done when each number in the dividend (the inside #) has a number on top of it for the answer.

Traditional Division:

Example: 347 ÷ 4

Step 1:
```
   0
 x
4 | 347
```
You can't make any groups of 4 out of 3.

Step 2: You can make 8 groups of 4 out of 34.
```
   08
 x
4 | 347
  -32⇩   ←(8 groups of 4 = 32)
   27    ←(You have 27 left over to ÷)
```

Step 3: You can make 6 groups of 4 out of 27.
```
   086
 x
4 | 347
  -30⇩
   27
  - 24   ←(6 groups of 4 = 24)
    3    ←(You have 3 left over to ÷)
```

Step 4: You can't make any groups of 4 out of 3, so 3 is your remainder (or left over).
```
   086 R3
 x
4 | 347
  -30⇩
   27
  -24
    3   ←(You have 3 left over)
```

Step 5: Answer is 86 with a remainder of 3 OR 86 ¾ or 86.75.

Partial-Quotients Division:

Example: 347 ÷ 4

Note: This method uses several different guesses as to how much will divide into the whole entire number instead of dividing small parts at a time. Students use extended facts (facts with 0's such as 50 x 4 = 200) instead of basic facts. Students would not write the **bold** parts on the left & right below, just the #'s.

Step 1: Ask yourself about how many groups of 4 can I make with 347? *(Students can take a random guess such as 20 or they can use reasoning such as follows... either way they should try to make a guess that will give an answer as close to 347 as they can. The answer to a guess cannot go over 347.)*

_: I know 4 x 100 is 400 and 347 is less than that so I will guess 80, OR 50 x 4 is 200 and 347 is a lot more than that so I'll guess 80.

```
                  4 | 347
80 groups of 4 =   -320  | 80 (Guess #1)
                    27   | ←(You have 27
                         |  left over to ÷)
```

Step 2: Ask yourself about how many groups of 4 can I make with 27?

```
                  4 | 347
80 groups of 4 =   -320  | 80 (Guess #1)
                    27   |
6 groups of 4 =    -24   | 6 (Guess #2)
                     3   | ←(You have 3
                         |  left over to ÷)
```

Step 3: Ask yourself about how many groups of 4 can I make with 3? You can't make any more groups so you are done. The left side is the remainder & you add your guesses (make sure they are lined up carefully) to get your answer.

```
                  4 | 347
80 groups of 4 =   -320  | 80 (Guess #1)
                    27   |
6 groups of 4 =    -24   | + 6 (Guess #2)
Remainder is 3           | 86  (or 86 ¾)
```

Partial-Quotients (Small Random Guesses):

Example: 347 ÷ 4

First Part: Ask yourself about how many groups of 4 can I make with 347? *(If students make random guesses they will still get the right answer but should still try to make large random guesses. Small guesses take more work... see next 2 examples)*

	4⟌347	
20 groups of 4 =	-80	20 (Guess #1)
	267	← (267 left to ÷)
20 groups of 4 =	-80	20 (Guess #2)
	187	← (187 left to ÷)
20 groups of 4 =	-80	20 (Guess #3)
	107	← (107 left to ÷)
20 groups of 4 =	-80	20 (Guess #4)
	27	← (27 left to ÷)
5 groups of 4 =	-20	5 (Guess #5)
	7	← (7 left to ÷)
1 group of 4 =	-4	+ 1 (Guess #6)
Remainder is 3		86 (or 86 ¾)

Last Part: Ask yourself about how many groups of 4 can I make with 3? You can't make any more groups so you are done. The left side is the remainder & you add your guesses (make sure they are lined up carefully) to get your answer.

Partial-Quotients (Big Random Guesses):

Example: 347 ÷ 4

First Part: Ask yourself about how many groups of 4 can I make with 347?

	4⟌347	
50 groups of 4 =	-200	50 (Guess #1)
	147	← (147 left to ÷)
40 groups of 4 =	-120	30 (Guess #2)
	27	← (27 left to ÷)
20 groups of 4 =	-24	+ 6 (Guess #3)
Remainder is 3		86 (or 86 ¾)

Last Part: Ask yourself about how many groups of 4 can I make with 3? You can't make any more groups so you are done. The left side is the remainder & you add your guesses (make sure they are lined up carefully) to get your answer. The bigger guesses you make, the less writing you have to do.

Partial-Quotients Division Mistakes:

Example: 347 ÷ 4

Note: The most common mistake with this method is simply errors in subtracting or not lining guesses up correctly to add when finished.

Ask yourself about how many groups of 4 can I make with 347?

	4⟌347	
100 groups of 4 =	-400	100 (Guess #1)

You can't subtract 400 from 347, the answer would be negative. If at any point in the problem your guess is too big, you must erase it and make a smaller guess.

Area Model Division: Example: 347 ÷ 4

Note: This method is very similar to the traditional method but uses place value to see how much will divide into the whole entire number with each step instead of thinking of just one digit. Students use extended facts (facts with 0's such as 50 x 4 = 200) instead of basic facts. Students would not write the bold parts on the left & right below, just the #'s.

Step 1: Start with the hundreds place and ask yourself about how many groups of 400 (4 hundreds) can I make with 347? I know 1 group would be 400 and 347 is less than that so the hundreds place must be a 0.

0 groups of 400 = x 0
4⟌347

Step 2: Now look at the tens place and ask yourself about how many groups of 40 (4 tens) can I make with 347? I know 8 groups would be 320 and 9 groups would be too much (360) and 347 is less than that so the tens place must be an 8.

0 groups of 400 = x 08
4⟌347
8 groups of 40 = -320
27 ←(You have 27 left over to ÷)

Step 3: Last look at the ones place and ask yourself about how many groups of 4 can I make with 27? I know 6 groups of 4 would be 24 and 7 groups would be too much (28) so the ones place must be a 6.

0 groups of 400 = x 086
4⟌347
8 groups of 40 = -320
27
6 groups of 4 = -24
3 ←(You have 3 left over to ÷)

Step 4: Ask yourself if you can make any more groups of 4? Since you can't make any more groups, you are done.

0 groups of 400 = x 086
4⟌347
8 groups of 40 = -320
27
6 groups of 4 = -24
Remainder is 3

Final Answer = 86 r3 (or 86 ¾ or 86.75)