❖ What is the process?

Step-by-Step Process for Model Drawing

1. Read the problem.
2. Rewrite the question in the problem as a complete sentence, leaving space for the answer.
3. Identify the *who* and the *what*.
4. Draw a unit for each variable.
5. Chunk the problem, and adjust the unit bars to match the information. Fill in the question mark.
6. Work your computation, and solve the problem correctly.
7. Write the answer in the sentence.

Sample Problem: Mr. Hobart sells 6 pans of brownies every day. He makes $10 per pan. How much money does Mr. Hobart make in a day?

Step 1 - Reading the Problem

No matter what way we solve a word problem, the first thing we do is always the same: We have to read it. Otherwise, we won't know what we're dealing with or who's involved or what we're supposed to do. Just saying, "Read the problem" doesn't give us a lot of direction for teaching model drawing to our students. So how do we have them read the problem? There are a few different techniques.

- We can write a problem on the board and ask the whole class to read it aloud together.
- We can ask each student to copy a problem from the board and read it to himself or herself silently.
- We can ask each student to read the problem from his or her own paper in a low voice.
- We can alternate calling on different students to read problems we put on the board.
- We can read a problem to students.
- We can have students work in partners and groups and have one partner read the problem to the others.

You might want to rely on one of these techniques more than others, but it's purely at your discretion. I personally like to mix it up, giving students a variety of ways to read the problems. Some kids respond really well to reading and hearing the problem at the same time. Others process it better quietly. By varying the approach, we give every student a better chance to learn.

Once we've read the problem, it's time to evaluate what we read. This is kind of like reading comprehension for math.

Step 2 – Rewrite the Question as a Complete Sentence (leaving a blank)

Mr. Hobart makes $_____ in a day.

Step 3 - Identifying the Variables

Our variables are the *who* and the *what* in the problem. Who does or has what, and how much of it? How does that relate to what the other person or people do or have? Believe it or not, figuring out the variables isn't usually too tricky, as long as students understand what the question is about. But as you can imagine, if this part is misunderstood or written down wrong, the whole problem collapses later on.

Let's revisit our problem with Mr. Hobart and his brownies. We'll peak into Mrs. Rutherford's classroom to see how she guides students to the variables.

Mrs. Rutherford: Okay, now that we've read the problem, it's time to find *who* and *what* we're talking about. Who is the word problem about?
Adam: Mr. Hobart?
Mrs. Rutherford: You're correct. Mr. Hobart. And is there anyone else?
Adam: No.
Mrs. Rutherford: Good job. So who knows the *what* for our problem?
Vanessa: Brownies.
Mrs. Rutherford: Does anyone else have another idea?
Lila: I think we're talking about Mr. Hobart's money.
Mrs. Rutherford: All right, I have a suggestion that will help you decide what it is we're solving for. Take a minute to read the last sentence in the word problem, and then tell your neighbor what you're solving for—brownies or money.
She gives them a minute.
Mrs. Rutherford: Okay. So what's the *what*?
Paulie: Money.
Mrs. Rutherford: That's correct. So our *who* is Mr. Hobart, and our *what* is money. Now let's set our problem up on the board here.

Did you notice how many probing questions Mrs. Rutherford asked her students? (*Is there anyone else? Is that what the problem is asking us?*) At this stage, these questions give students the chance to stop and consider whether they're missing anything. (As a quick side note, keep in mind that many problems have a *who* and a *what*, but some problems will only have a *who* or only a *what*. If the problem contains fewer variables, it just makes the process easier.)

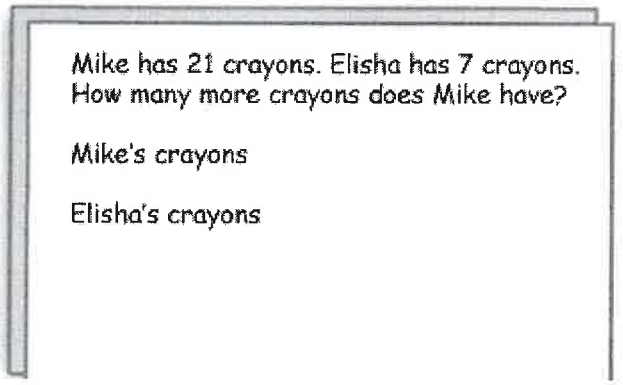
Now it's time to take the variable information and set our problem up. As you've probably noticed, model drawing sets up problems a little differently than we're used to, **with horizontal writing instead of vertical math**.

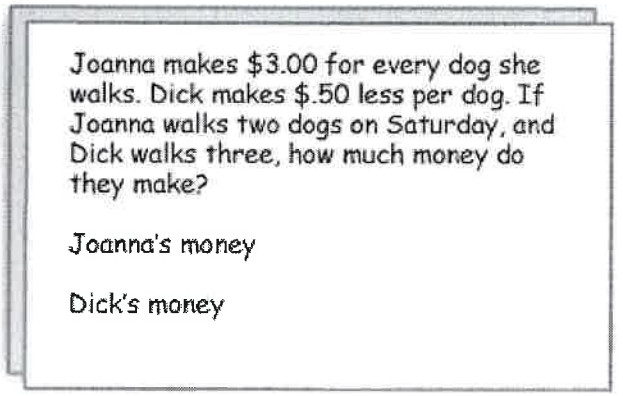
Setting Up Our Problem

Take a horizontal sheet of paper, and make sure the problem is written at the top of it. Then write your variables on the far left side of the page. As you determine a variable in the problem, you may circle or underline it in the word problem itself. Later on, this can remind you of your variables. But that's just an optional step—if you like it, use it. And if you have visual students, encourage them to use it, too.

If there are multiple variables, write each in the order in which it appears in the problem. For example, if the problem mentions Mike first and then Elisha, put Mike before Elisha as you list variables one under the other.

Here are some examples of ways to write variables.





So let's see how Mrs. Rutherford and her class write the variables.

> Mr. Hobart sells 6 pans of brownies every day. He makes $10 per pan. How much money does Mr. Hobart make in a day?
>
> Mr. Hobart's money

Once the variables are written down, it's time to add the unit bar or bars directly to their right.

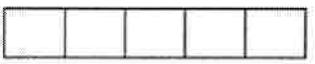
Step 4 - Drawing Unit Bars

Unit bars give us a way to represent how much money Mr. Hobart makes. They provide the visual that complements our math so well. In early grades, and with simple math problems that have low numbers, we can let students draw individual units (like five fish) or use manipulatives to represent the *who* in the problem.

But as a general rule, we want students to use neatly drawn squares or rectangles of the same size for each unit bar. These keep the drawings uniform and give us a baseline. When students in the primary grades learn model drawing, the unit is small and the value is worth one. When the numbers in the word problem get larger, the units will be larger, and the value can be any number inside.

Here are a few examples:

One-to-One Correspondence Part-Whole or Comparison Model

> Mike picked 5 apples and 3 oranges.
> How many pieces of fruit did Mike pick?
>
> Mike's apples 
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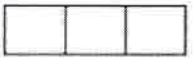
> Mike's orange 
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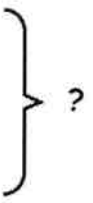
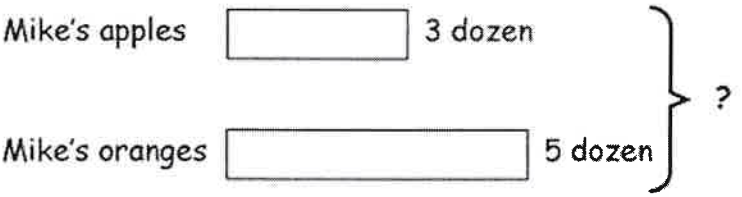
> 
>
>
> Mike picked _____ pieces of fruit.

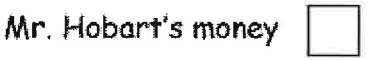
Part-Whole or Comparison With Larger Numbers

Mike picked 3 dozen apples and 5 dozen oranges.
How many pieces of fruit did Mike pick?



Mike picked _____ pieces of fruit.

Remember that this is just the first time we're using our unit bars (we'll adjust them next), and they need to be a uniform size in the beginning. (Of course, if you have just one unit bar for a problem, it can be whatever size you want it to be.) Here's Mr. Hobart's unit bar (below). I made it on the small side—not because the value means one item, but because I know he has several groups of money, and we'll be adding more to this original unit.



Why is this so important? Because the essence of the word problem is that it asks us to figure out the correlation between two related things, like how much money Mr. Hobart makes per pan and how much he makes total in one day. The key to that sentence is *between two related things*. Word problems give us the relationship, but we have to draw it so that we show that equality up front. When it's time to adjust the unit bars to chunk information, everything changes.

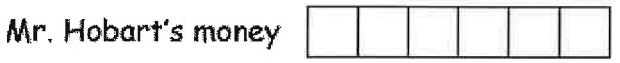
Step 5 - Rereading Each Sentence and Adjusting the Unit Bars

The next step is to reread the problem one sentence at a time, chunking information to make it manageable. Then we adjust the unit bar or bars to match the information in the problem. This is a step where it's easy to make a mistake, so let's walk through it slowly.

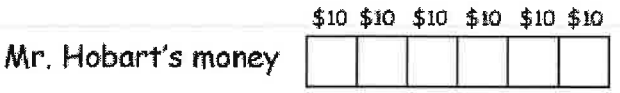
The first thing we need to do is chunk our information, rereading each sentence. Let's look at our problem again. Example:

Mrs. Rutherford: Okay, class. We know that our *who* is Mr. Hobart, and our *what* is his money. We have our unit bar, and now it's time to reread the problem and adjust our unit bar so it matches the information we have in the problem.

Mrs. Rutherford: So what do we know about Mr. Hobart by reading our first sentence?
Alex: He sells 6 pans of brownies every day.
Mrs. Rutherford: Good job. So let's show his 6 pans. We have one now, so how do we turn one into 6?
Jim: By adding 6.
Mrs. Rutherford: If we add 6, won't we have 7 pans?
Carol: We have to add 5.
Mrs. Rutherford: Let's try it and then look at our model to make sure it mirrors our words. Please read the sentence again and count how many units you have. Does your picture tell the story of the word problem?



Mrs. Rutherford: So now we've represented our 6 pans. What does the next sentence tell us?
Katy: That he makes $10 per pan.
Mrs. Rutherford: Great job. You're correct. So how do we adjust our bar to explain that?
Vivian: Can we mark each piece with $10?
Mrs. Rutherford: Let's try that out.



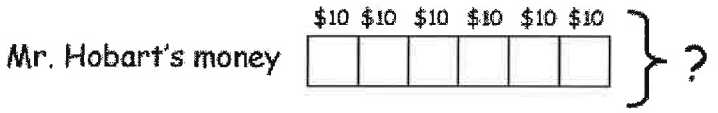
Mrs. Rutherford: I'm thrilled with how our model looks. Please tell your neighbor how many pans of brownies Mr. Hobart has, and how much each unit is worth.

See—that wasn't too hard, was it? As you're adjusting the unit bars, you'll want to write the appropriate quantities either right above or below the bars in grades three and up, or right inside in the unit bars in grades one and two. The important thing is to represent each relevant quantity so it's clearly marked.

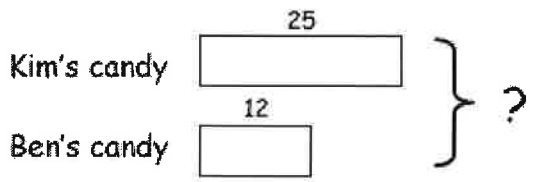
Adding the Question Mark

As you can probably imagine, we add our question mark to indicate what we need to find out and where. Sometimes it surrounds multiple unit bars with a big bracket, and other times, it just relates to one part of one unit bar with a small bracket or no bracket at all. Let's see how Mrs. Rutherford's class adds the question mark to Mr. Hobart's unit bar.

Mrs. Rutherford: Now we need to address the big question: What are we trying to figure out? That means we're ready to place our question mark. So what do we need to know?
Cindy: How much money Mr. Hobart makes per day with his brownies.
Mrs. Rutherford: Are we solving for the part or whole that Mr. Hobart earns?
Ethan: We're solving for all of the money Mr. Hobart makes.
Mrs. Rutherford: Good. So where shall we place our question mark?
Mark: At the end of the unit bar.
Mrs. Rutherford: Yes. And we'll put a little bracket there to clarify that we mean the total of the unit bar.



There's something important we need to notice here, and it's that we have to use those brackets to clarify what all our question mark is referencing. Sometimes the question mark represents the combined value of two unit bars, so in that case, the bracket would go around both. Here's an example.

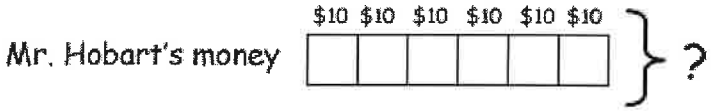


Step 6 - Working the Computation

Now, you'll probably remember that you have some choice about where you solve the computation. My preference is to do it to the right of the question mark and unit bars, but space is sometimes an issue there on the right side of the page. So you can always solve the computation underneath the unit bars if that feels more comfortable.

Let's take a look at how Mrs. Rutherford's class solves the question of Mr. Hobart's money.

Mrs. Rutherford: Now it's time to write our computation. So we want to figure out what six $10 profits equal, as we established last time. What would that look like?
Mike: 6 x 10!
Mrs. Rutherford: Good job. Let's step it out.



1 unit = $10

6 units = ?

10 X 6 = 60

or alternatively

10 + 10 + 10 + 10 + 10 + 10 = 60

$60

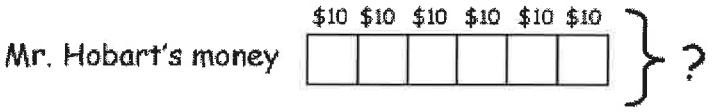
Mrs. Rutherford: As you can see, we can either use the multiplication like Mike suggested, or we can add $10 six times like Sally suggested, or we can do it the way Willy showed us. It's important to use the strategy that makes the most sense to you. It won't always look like your neighbor's! One of my goals is that you're able to explain what your model represents and how you solved for the answer.

Did you notice how we start with what we know in our computation (1 unit = $10)? Then we put our question mark in written form (6 units = ?). Finally, we solve the math (6 x 10 or 10 + 10 + 10 + 10 + 10 + 10). The total: 60. And let's remember that we're talking about money, so it's $60. Now it's time for the final step: writing a complete and grammatically correct sentence.

Step 7 - Writing the Sentence

This last step is the one that's easiest to forget, because once we arrive at the answer ($60), our tendency is to move on to the next problem rather than reiterate our answer in proper English. But filling in the answer in the sentence is important, because it brings the learning full circle.

Here's the sentence for Mr. Hobart's brownies:



1 unit = $10

6 units = ?

10 X 6 = 60

or alternatively

10 + 10 + 10 + 10 + 10 + 10 = 60

$60

Mr. Hobart makes $60 in a day.