❖ What Is Singapore Math?

Singapore Math is a base-10 math program that forms the backbone of math instruction in Singapore. Yes, it all began thousands of miles from here on the southern tip of the Malay Peninsula. **Singaporean students far exceed any other nation's students in math achievements. And they've set the record in math performance for over a decade and a half. Their students are proactive problem solvers who can easily do in third grade the same problem many of our students struggle with in sixth grade.**

So some of us over here in the U.S. started asking, "Why?" After all, holding the record for math achievement is a pretty tremendous accomplishment. "Why?" led to "What?" and "How?" and after that, we started rooting around for some answers. If students in Singapore have such tremendous success learning math a certain way, can't we replicate that success here? Yes! Let's take a quick tour of the Singapore curriculum to see how it works and how model drawing fits in. There are three pieces that form the backbone of Singapore Math instruction, and believe it or not, they're all pretty easy to implement once you get used to them.

1.) Problem Solving

Problem solving is one of the three main components of Singapore Math. I meet a lot of teachers who think model drawing represents the entire problem-solving component of Singapore Math, but that's simply not the case. It's but one critical piece. The idea is that the models give students a tangible graphic to hold onto as they compute a word problem. As such, most math computations become candidates for model drawing. So if you have some students who have trouble understanding what math means, you'll find that the models make things a lot clearer. Model drawing is the essential tool taught and used to solve word problems in Singapore.

For problems that aren't candidates for model drawing, students in Singapore use other heuristics, or problem-solving strategies, just like we do. They make charts, act out the problems, draw tables, use before-and-after concept strategies, look for patterns, and guess and check their work. The goal and heartbeat of the education framework in Singapore is to develop students into problem solvers. And model drawing is one way to accomplish this goal.

2.) Intense Number Sense and Place Value Instruction

In addition to problem solving, Singapore Math focuses intensely on number sense and place value—two concepts that go hand in hand. *Number sense* is a student's overall understanding of a number, and *place value* is a student's understanding of a digit's value position in a number. Why are they important?

Because they help students learn what 10 means, for example, instead of just memorizing that 10 is 10. That may not sound very important when we're just talking about plain old 10, but when we start talking about 4,912 or 1,005,439, it becomes really important.

Think of it this way. Let's say I ask you to memorize a recipe for making green paint. To do this, you take one part blue paint and one part yellow paint and stir. Okay? It sounds pretty simple, kind of like knowing that 10 is 10. It's easy to remember. But what if instead of telling you how to make green paint, I explain the concept behind primary colors and how they blend to form other colors? Then you learn about making purple, turquoise, magenta, sage, and all sorts of other colors. This helps you understand color combining, right?

So, effectively, Singapore Math teaches color combining instead of just recipes for one shade of paint. Students then have a better base for understanding how fractions are parts of wholes and decimals embody the same concept in a different form. Word problems make more sense because they make sense in the greater context of numbers in general.

3.) Mental Math

Singapore Math also gives students training in *mental math* so they can transition from doing math on paper to doing it automatically in their minds. What's the benefit? Solving math problems becomes much faster and easier. With mental math drills, students practice computational strategies until they transfer those skills from a paper-and-pencil drill to a mental skill.

You'd be amazed by the types of computations Singaporean students can do in their heads. For example, fourth grade students can easily solve 125 x 6 without using a pencil. Can you do that? And there's a strategy for solving mental math problems: breaking numbers into manageable pieces using the distributive property. Let's see how it works with 125 x 6. We'll start with the big chunks, and you'll notice that we'll decompose the multidigit number into place value groups.

Mental Math Drill
(100 x 6) + (20 x 6) + (5 x 6)
Then, we compute our parts.
(600 + 120 + 30)
Next, we bring together the parts to get our whole.
(700 + 50)
That's a total of 750.

Pretty impressive, and it only takes students a few seconds! When we put together the three components of Singapore Math (problem solving; number sense and place value instruction; and mental math), we have the curriculum that the world's math leaders use!

❖ What Is Model Drawing?

Model drawing, or MD, is a step-by-step visual method of turning a word problem into a diagram with unit bars that represent values. Just the mention of "word problems" is enough to scare some students into permanent mathophobia, but let me assure you that model drawing is easy enough to coax them out of these apprehensions and into the world of math. Let's take a look.

Mrs. Jenkins has given her second-graders the following problem:

> Mary has 4 fish. Connie has 7 fish. How many fish do Mary and Connie have altogether?

How do we solve this problem in our classrooms right now? Chances are good that we ask students to align 4 and 7 vertically, put a line under the numbers, and find 11 as the answer.

$$\begin{array}{r} 4 \\ +\ 7 \\ \hline 11 \end{array}$$

That's all well and good if our students know how to extract the 4 and the 7 from the problem and make their equation. But what if the word problem was more complex? What if Mrs. Jenkins was teaching fifth-graders and the problem was like this one?:

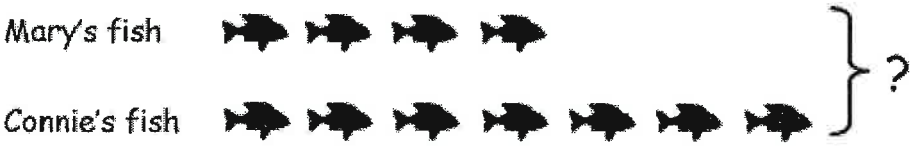
> Ignacio can type 62 words per minute. Carol types 82 words per minute. If they both type for seven minutes, what is the average amount of words they'll produce?

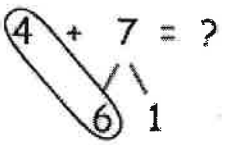
Yikes. This is the kind of problem where we lose everybody before we even get started. Our students suddenly get very interested in the eraser fuzz on their desks. Why? Kids don't know what equation they're meant to solve. Are they supposed to figure out words per minute, or how much Ignacio can type—or what? **In *traditional algorithm math*, the math most of us use in the classroom, there isn't a structure that gives students a framework for solving all types of word problems in the same way.**

Now I bet you're thinking, "But you can't solve all types of word problems in the same way." Oh, I beg to differ. Model drawing offers an easy alternative with a proven process that works equally well for first-grade addition or sixth-grade pre-algebra. Same process, different numbers, different names.

Let's see how we can use model drawing to figure out our fish.

Mary has 4 fish. Connie has 7 fish. How many fish do Mary and Connie have altogether?





10 + 1 = 11

Mary and Connie have 11 fish altogether.

While this may not look very sophisticated at the moment, there's a step-by-step process at work in the way we solve that problem.

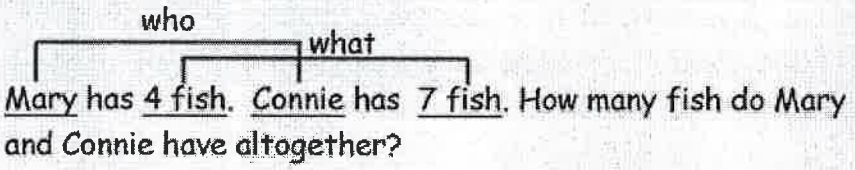
Step-by-Step Model Drawing

1. Read the problem.

2. Rewrite the question in the problem as a complete sentence, leaving space for the answer.

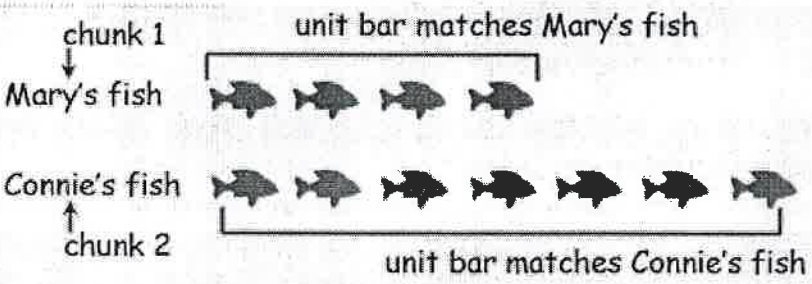
   Mary and Connie have _____ fish altogether.

3. Identify the *variables*, which are the *who* and the *what*. Write them down in the order they appear in the problem. Do this on the far left side of the paper or the board.

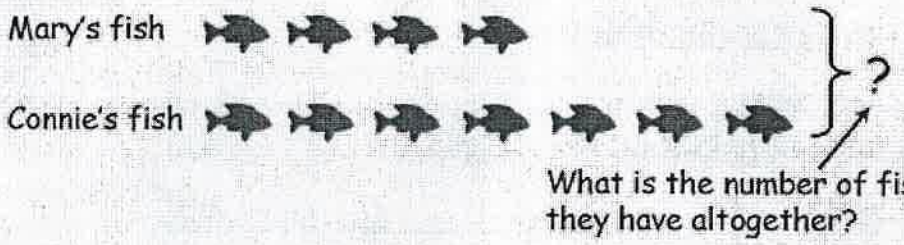


4. Draw a *unit bar*, or a rectangular representation of each variable. This (or these if you have more than one unit bar) goes immediately to the right of the variables. In younger grades, this unit bar might include little fishes. In older grades, it's simply a rectangle or square that gets added to, subtracted from, or divided.

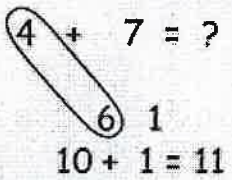
5. Reread the problem one sentence at a time, chunking information to make it manageable. Adjust the unit bar or bars to match the information.



6. Decide on your *question mark*, or what the problem's asking you for, and draw it in the appropriate place.



7. Work the computation. You can work the computation to the right of the unit bars or beneath the unit bars, depending on your preference or on the space on the page. **Note:** I recommend that you do the math horizontally, regrouping numbers into units of 10 wherever possible. That's what I'll be demonstrating in this course. But you can also do vertical math and skip the regrouping.



8. Write a complete and grammatically correct sentence to answer your question mark. This goes beneath all your work.

9. Write the answer in your sentence.

Mary and Connie have 11 fish altogether.

This step-by-step process is based on a strategy developed by George Polya: How to Solve It. Here's what it looks like:

1. **See.** What is the problem asking us to do? What are we trying to find out? Can we restate the problem?

2. **Plan.** What do we know? What do we need to do to solve the problem? Do we need more information? Is there a hidden question? What strategies are useful?

3. **Do.** Carry out the plan. Apply mathematical skills, concepts, and strategies.

4. **Check.** Compare with the original question. Does the answer make sense? Should we revise our plan to meet all of the conditions?

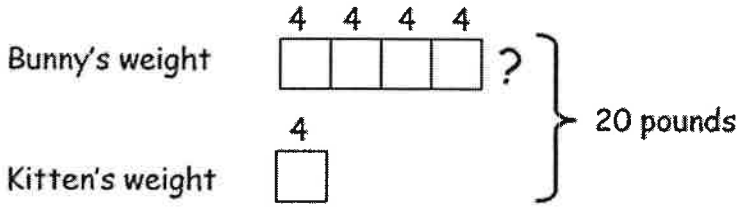
❖ **The Benefits of Model Drawing**

By now you're probably thinking, "Well, this sounds great, but what does it mean for me?". MD offers a totally new way of looking at numbers, and it's a lot of work to think about and **teach math totally differently**. However, it's all worth it because you'll quickly notice a wonderful transformation within your students. What changes? They stop being intimidated by math and start enjoying it. No, really! Let's look at some of the ways MD will help your students make pleasant (and often new) connections with numbers.

- **Students have one strategy for solving most word problems:** One of the main benefits of model drawing is that students can solve a number of different word problems with the same step-by-step process. It doesn't matter whether the problem is about miles per hour or multiplying minutes. **The process works with about 80% of the word problems out there in elementary school.**

  Take a look at these two problems to see what I mean:

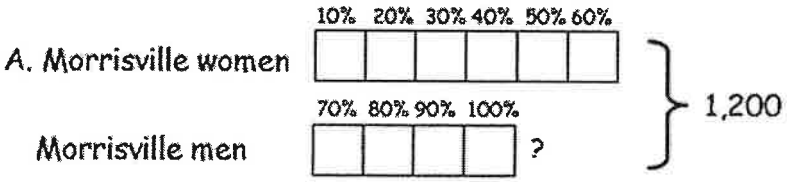
A bunny weighs 4 times as much as a kitten. If their total weight is 20 pounds, what is the total weight of the bunny?



A. 5 units = 20 pounds
1 unit = 20 ÷ 5 :
= 4 pounds
4 units = 4 X 4
= 16 pounds

The bunny weighs 16 pounds.

In Morrisville, there are 1,200 residents. 60% of them are women.
Find out how many men are in Morrisville.



B. 100% = 1,200
10% = ?
1200 ÷ 10 = 120
10% = 120

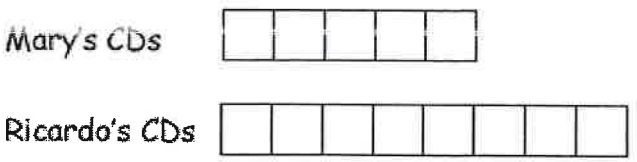
C. 10% = 120
40% = ?
120 X 4
(100 X 4) + (20 X 4)
400 + 80 = 480 men

There are 480 men in Morrisville.

These problems are very different, but we use the same setup for both. Doesn't it sound easier to master this process and use it over and over again instead of solving addition word problems one way, decimal word problems another, and fraction word problems a third way?

- **Students have a visual to associate with numbers that can be abstract:** There's something about the unit bars that helps students visualize what can often be abstract math concepts. For example, what does it mean if Mary has 5/8 the CDs that Ricardo has? If you try to picture that in your mind, chances are good that nothing really clicks.

  That's because it's hard to visualize 5/8 of a CD collection. No wonder students have trouble with fraction word problems! See, 5/8 of something is an abstract concept that doesn't hold a lot of meaning in our daily lives. But unit bars give us something to hold onto. With them, we can draw the problem so it does make sense.



- **Students learn to translate the English into math and then back into English:** Have you ever been frustrated by math? I'm raising my hand! Sometimes it's hard to understand why working with x and y and halves and wholes is even worth our time. Guess what? Students think the very same thing. Why does this matter? When will I ever need to know how many men live in Morrisville? Furthermore, who cares?

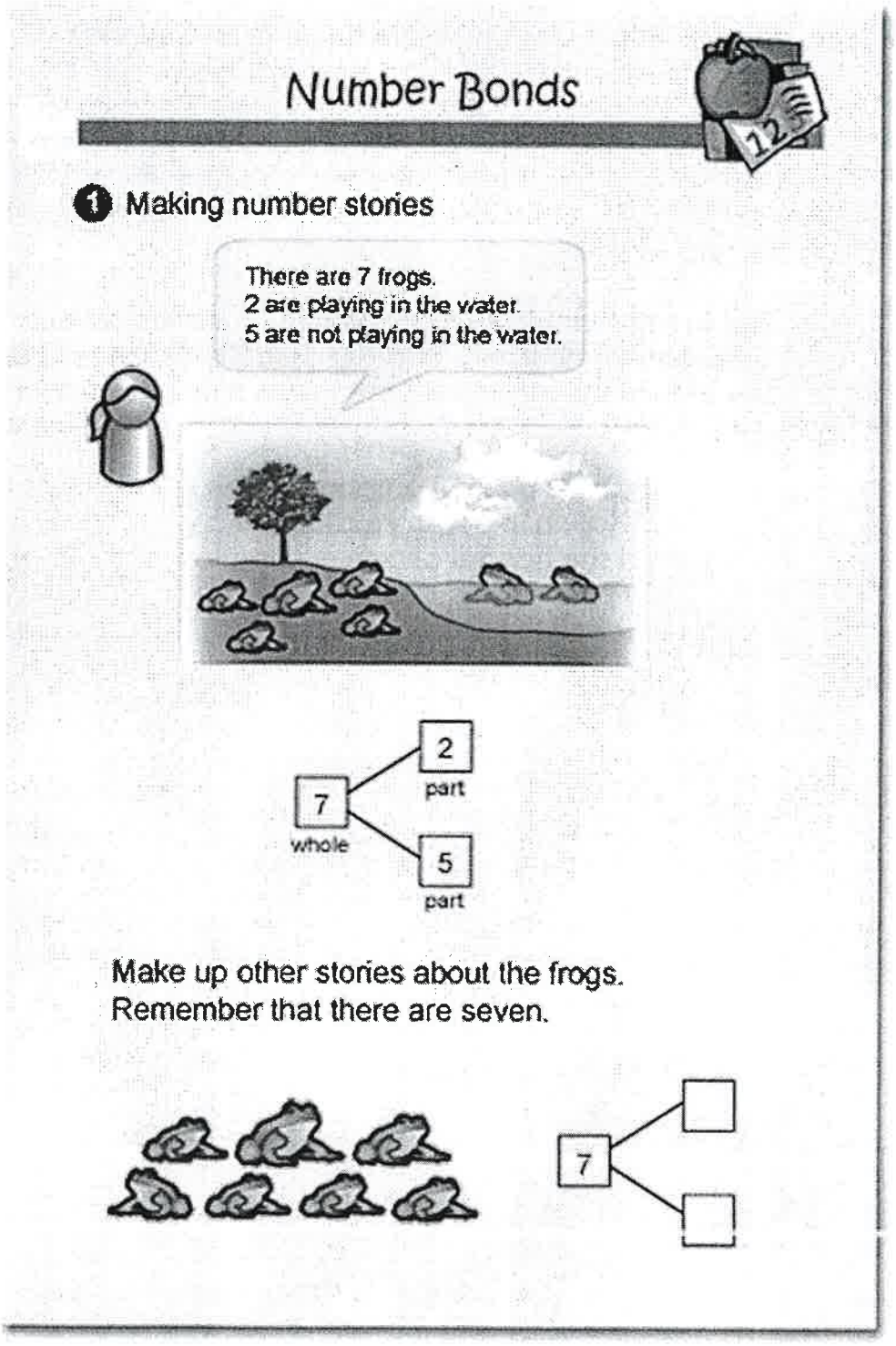
  Usually, students strip a word problem down to an equation very quickly, and they lose the meaning. The word problem seems only like a vehicle for delivering a computation. But not with model drawing! MD builds a bridge between everyday scenarios (word problems), the equations to solve them in context (math), and the solutions (real-life answers in complete sentences). What could be better than helping children relate math to the real world?

- **Students start to see the relationships behind numerical values:** When numbers just seem like numbers, and word problems like problems, math loses its relevance in the classroom. Math is so much more than just a collection of numbers, but it's hard to help students see it that way.

  One of MD's strong points is that it emphasizes the relationships between values in the computation. It doesn't matter if Mary has 5/8 of something if we don't know what she has 5/8 of or in relation to whom. By emphasizing all parts of the word problem equally (and giving each one a visual), we teach students to look at the problem for the way the parts relate to the whole.

Kindergarten – 2nd Grade

We start with picture-rich stories in kindergarten & first grade. Over the first half of 1st grade, we have students fill in stories with *number bonds*, or number groupings that equal 10. Here's an example:



In the second half of 1st grade, we have students fill in number sentences for word stories. Here's a sample.

Write a number sentence for this story.

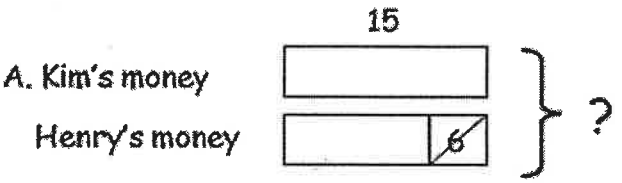
5 turtles 

3 squirrels 

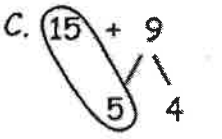
There are _____ more turtles than squirrels.

Second grade is when we get into the real meat of model drawing, starting students on basic word problems that they will continue to solve through sixth grade and beyond. **Second grade is where we start to teach the process in its full form.**

Kim earned $15 dollars by raking leaves. That was $6 more than Henry earned. How much did they make altogether?

A. 

B. 15 - 6
/ \
10 5
10 - 6 = 4
4 + 5 = $9 for Henry.

C. 

$20 + $4

D. 10 + 4 + 10 = $24

Kim and Henry have $24 altogether.